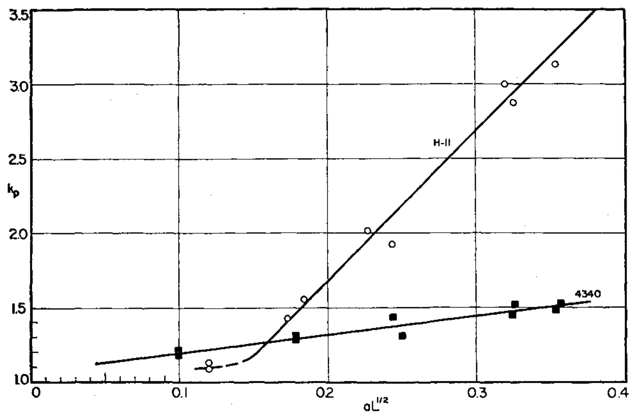

Fig. 5 Test data on 0.070-in.-thick sheet specimens of H-11 and 4340

Manning's data[7] on 4340 and H-11 steels. Both steels had the same yield strength of 225 ksi with an ultimate tensile strength of 260 ksi for 4340 and 270 ksi for H-11. The tests were conducted on specimens 1.5-in. wide with fatigue cracks emanating from a central circular hole that was varied in radius. It can be observed that the test data correlate reasonably well with the straight lines shown in Fig. 5.

In this case, the crossover point occurs at a value of $aL^{1/2} = 0.16$, which corresponds to a crack length of 0.025 in. ($\frac{1}{3}$ of the thickness) in a structure. The crossover point in both Figs. 4 and 5 appears to be associated with the effects of plasticity at the lower values of k_p. As discussed in relation to Eq. (6), a linear relation between k_e and k_p can be expected when the net section stress is elastic. At low values of k_p the net section stress is in the plastic range, and the departure from a straight line shown for the lowest H-11 data points in Fig. 5 is to be expected.

Equation (6) indicates that the slope of the straight line through the test data is a measure of the crack-ductility ratio $e/r^{1/2}$. The values shown in Table 1 were obtained from Figs. 3–5.

Conclusions

1) From the test data presented herein, it appears that the use of Eq. (6) is satisfactory for correlating test data on the residual strength of fatigue cracked specimens of varying widths.

2) This correlation scheme indicates that, although higher strength materials may be more notch sensitive for larger crack lengths, they also may be less sensitive for small crack lengths than lower strength materials. This phenomenon appears to be associated with the effects of plasticity.

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Table 1 Summary of correlated test data

Material	σ_{tu}, ksi	σ_{ty}, ksi	$e/r^{1/2}$, in.$^{-1/2}$
2024-T3	72	53	0.23
7075-T6	80.5	74	0.89
4340	260	225	0.63
H-11	270	225	4.95

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Shape of the Porous Two-Dimensional Hypersonic Sail

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In an earlier note[1] the author pointed out how one may calculate the effect of porosity on the characteristics of the two-dimensional supersonic sail.[2] (M. R. Fink, United Aircraft Corporation, has drawn the author's attention to a similar study of the impermeable supersonic sail he published in an internal company report.[3]) Here the effect of porosity on the shape is calculated, and hence the modification to the aerodynamic performance of the impermeable hypersonic sail studied by Daskin and Feldman[4] and by the author.[5]

First establish the loading on the porous sail in a Newtonian flow (see Fig. 1 for notation). A thin shock layer forms on the concave forward-facing surface of the sail. Define P such that $\rho_\infty U^2 P$ is the momentum flow in the shock layer per unit depth. Assume that the velocity along a streamline within the shock layer is constant. For a streamline entering the shock layer where the inclination is θ' this velocity is $U \cos\theta'$. Thus, since the differential element of mass flow per unit depth is $\rho_\infty U\, dy'$, the value of P at y is

$$P = \int_0^y \cos\theta'\, dy' \qquad (1)$$

Because of the porosity there is a normal velocity v through the sail, where one may write

$$v/u = \sigma(p)(p/q_\infty) \qquad (2)$$

Here p is the pressure difference across the sail, and σ is a parameter describing the porosity.

The drag of that part of the sail between $y = 0$ and y is calculated most easily from a momentum balance. The drag per unit depth

$$D = \rho_\infty U^2 y - \rho_\infty U^2 P \cos\theta - \rho_\infty U \int_0^y v \sin\theta'\, dy' \qquad (3)$$

and the drag coefficient for unit chord

$$C_D = 2y - 2P\cos\theta - 2\int_0^y \frac{v}{U}\sin\theta'\, dy' \qquad (4)$$

The pressure difference $p(y)$ across the sail is given by

$$p/q_\infty = dC_D/dy = 2\sin^2\theta + 2\sin\theta\frac{d\theta}{dy}\int_0^y \cos\theta'\, dy' - 2\sigma(p)\frac{p}{q_\infty}\sin\theta \qquad (5)$$

$$(p/q_\infty)[1 + 2\sigma(p)\sin\theta] = 2\sin^2\theta + 2\sin\theta\frac{d\theta}{dy}\int_0^y \cos\theta'\, dy' \qquad (6)$$

The shape of the sail is found by substituting this loading

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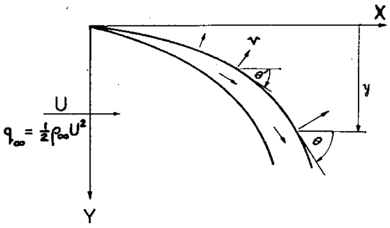

Fig. 1 Sail geometry and notation

in the equilibrium equation of the sail

$$p = T(d\theta/ds) \tag{7}$$

where s is the arc length of the sail and T the tension in the sail per unit depth.

First neglect the centrifugal correction and correct the shape of the sail studied by Daskin and Feldman.[4] For simplicity assume that σ is independent of ρ. Thus one gets immediately

$$\frac{2q_\infty}{T} s = \int_{\theta_L}^{\theta} \frac{d\theta}{\sin^2\theta} + 2\sigma \int_{\theta_L}^{\theta} \frac{d\theta}{\sin\theta} = (\cot\theta_L - \cot\theta) + 2\sigma \ln(\tan\tfrac{1}{2}\theta/\tan\tfrac{1}{2}\theta_L) \tag{8}$$

$$\frac{2q_\infty}{T} x = \int_{\theta_L}^{\theta} \cos\theta \frac{2q_\infty}{T}\frac{ds}{d\theta} d\theta = \left(\frac{1}{\sin\theta_L} - \frac{1}{\sin\theta}\right) + 2\sigma \ln(\sin\theta/\sin\theta_L) \tag{9}$$

$$\frac{2q_\infty y}{T} = \int_{\theta_L}^{\theta} \sin\theta \frac{2q_\infty}{T}\frac{ds}{d\theta} d\theta = \ln(\tan\tfrac{1}{2}\theta/\tan\tfrac{1}{2}\theta_L) + 2\sigma(\theta - \theta_L) \tag{10}$$

If one retains the centrifugal term, one must solve the integrodifferential equation

$$\frac{ds}{d\theta}\sin^2\theta + \int_{\theta_L}^{\theta} \cos\theta' \sin\theta' \frac{ds}{d\theta'} d\theta' = (T/2q_\infty)(1 + 2\sigma\sin\theta) \tag{11}$$

Putting $\phi(\theta) = (ds/d\theta)\sin^2\theta$ and differentiating Eq. (11), one finds

$$\phi'(\theta) + \cot\theta\,\phi(\theta) = (T/2q_\infty)2\sigma\cos\theta \tag{12}$$

which has the solution

$$\phi(\theta)\sin\theta = \frac{T}{2q_\infty}\sin\theta_L(1 + \sigma\sin\theta_L) + \sigma\frac{T}{2q_\infty}\sin^2\theta \tag{13}$$

for, from Eq. (11), when $\theta = \theta_L$, $\phi(\theta_L) = (T/2q_\infty)(1 + 2\sigma\cdot\sin\theta_L)$.

Thus one finds

$$\frac{2q_\infty}{T} s = \sin\theta_L(1 + \sigma\sin\theta_L)\int_{\theta_L}^{\theta}\frac{d\theta}{\sin^3\theta} + \sigma\int_{\theta_L}^{\theta}\frac{d\theta}{\sin\theta} = \tfrac{1}{2}(1 + \sigma\sin\theta_L)\{\cot\theta_L - \cot\theta(\sin\theta_L/\sin\theta) + \sin\theta_L \ln(\tan\tfrac{1}{2}\theta/\tan\tfrac{1}{2}\theta_L)\} + \sigma\ln(\tan\tfrac{1}{2}\theta/\tan\tfrac{1}{2}\theta_L) \tag{14}$$

$$2q_\infty x/T = \tfrac{1}{2}(1 + \sigma\sin\theta_L)[(1/\sin\theta_L) - (1/\sin\theta)\cdot(\sin\theta_L/\sin\theta)] + \sigma\ln(\sin\theta/\sin\theta_L) \tag{15}$$

$$2q_\infty y/T = (1 + \sigma\sin\theta_L)[\cos\theta_L - \cos\theta(\sin\theta_L/\sin\theta)] + \sigma(\theta - \theta_L) \tag{16}$$

The aerodynamic forces and moments on the sail follow immediately from resolving and taking moments of the attaching forces, and are simply expressed in terms of the sail geometry.[2–5]

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Mechanism of the Accelerated Burning of Ammonium Perchlorate at High Pressures

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The linear burning rate of ammonium perchlorate previously has been shown to undergo a marked increase in pressure dependence at high pressures. The effect was considered to result from an increased burning surface area formed by a shear process at the burning surface as a result of the high pressure. The possibility that the steep thermal gradient existing at the burning surface at high pressures also could lead to the shear stress responsible for cracking is investigated. The analysis indicates that thermal stress is almost solely responsible for the cracking over the entire pressure range of the burning rate experiments.

The deflagration characteristics of pure ammonium perchlorate (AP) strands previously have been investigated by means of a closed-bomb strand burning technique at pressures from 1000 to 23,000 psi.[1] The data are in general agreement with vented-chamber AP burning-rate data of other investigators at pressures from 1000 to 5000 psi. At pressures above 5000 psi (the pressure limit of previously reported studies), a marked increase in pressure dependence of the linear burning rate occurs.

The observed increase in burning rate was considered to result from an increased burning surface area produced by the action of the very high pressures in the closed bomb.[1] It was postulated that the pressure (i.e., stress) acting upon the nonhomogeneous burning surface caused shearing that gave rise to increased burning area by forming new cracks and pores or by enlarging existing cracks and pores. The crack growth process was analyzed in terms of the Eyring theory of creep and fracture. A geometrical model was presented which considers the accelerated burning process as a development of micro-cracks that form into conically shaped burning surfaces, the area of which depends upon the pressure. The model was in good agreement with the experimental burning-rate data and with the pressure vs time data for individual burning-rate experiments at pressures above 5000 psi.

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